Modelling and numerical simulation in cardiac electrophysiology: reduced order modelling and inverse problem

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Abstract

The electric wave in the heart is gouverned by a system of reaction-diffusion partial differential equations called the bidomain model. This system is coupled nonlinearly to an ordinary differential equations (ODEs) modeling the cellular membrane dynamics. The bidomain model is widely used in cardiac electrophysiology simulation. This mathematical model takes into account the electrical properties of the cardiac muscle. Different numerical methods have been used for solving the bidomain model. Finite element method, finite difference method and finite volume method. All these methods lead to a large linear system to solve, especially when using implicit schemes. The numerical cost in this case becomes very important when we are interested in solving inverse problems. The computational cost of solving the bidomain problem becomes very important when we are interested in solving inverse problem. Thus reducing the computational cost of the forward problem is a challenging issue. One of the most popular approaches used in model reduction is the *Proper Orthogonal Decomposition* method. This method was initially introduced for analyzing multidimensional data.

In this work we present a reduced order approach based on POD method for the computation of the electrical activity of the heart. We stability result of the POD method based on an a priori error estimate. This theoretical result shows that we can control the gap between the full finite element solution and the POD solution of the bidomain equation by controlling the gap between the finite element solution and its projection on the used POD basis. We also show that the POD method could be used for different strategies of solving the bidomain model. It could be used for a fully coupled scheme or by using a time splitting schemes. The numerical results show that it is stable in both cases. In order to evaluate the usefulness of this approach in parameter estimation problem, we build a POD basis using the original parameters of the ionic model and we computed the L^2 relative error between the finite elements solution and the reduced order solution for different parameters. Also we conclude that in case of parameter estimation framework it is recommended to use the POD in order to estimate τ_{close}, τ_{open} and τ_{out} introduced in the formula of the ionic current model. But to estimate the parameter τ_{in}, the data from which the POD basis is computed should be sufficiently rich in order to maintain a good accuracy of the results.

In the second part of this work, we will intererst to established a theorical stability estimates for the parameter identification τ_{in} to which the solution is the most sensitive. We established a new Carleman inequality for a reaction diffusion equation coupled to an ordinary differential equation. The Carleman inequality that we established for the ODE was fundamental in order to prove the global Carleman estimate for non linear parabolic equation coupled with an ordinary differential equation and solving the parameter stability problem.

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