An approach to computer-assisted existence proofs for nonlinear space-time fractional parabolic problems

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Abstract

Let $\Omega \subset \mathbb{R}^n$ be a bounded convex domain with polyhedral boundary. Given $s \in (0,1)$ and $\gamma \in (0,1]$ we consider an initial boundary value problem for the following space-time fractional parabolic equation:

$$\begin{cases} \partial_t^\gamma u(x,t) + (-\Delta)^s u(x,t) &= f(x,t,u) \quad \text{in } \Omega \times (0,T), \\ u(x,0) &= u_0(x), \quad x \in \Omega, \\ u(x,t) &= 0 \quad \text{on } \partial\Omega \times (0,T), \end{cases} \tag{1}$$

where the fractional derivative in time ∂_t^γ for $\gamma \in (0,1)$ is understood as the Caputo fractional derivative of order γ with respect to t, which is defined by

$$\partial_t^\gamma u(x,t) := \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{1}{(t-\eta)^\gamma} \frac{\partial u(x,\eta)}{\partial \eta} d\eta.$$

For $\gamma \to 1$, this converges to $\partial_t u(x,t)$, if $\partial_t u(x,\cdot)$ is continuous. The fractional Laplacian $(-\Delta)^s$ is defined in the spectral sense as explained in the talk. Instead of dealing with the nonlocal operator $(-\Delta)^s$ of problem (1) we treat a corresponding local problem which is obtained by Caffarelli-Solvestre extension technique, and show how to enclose a solution of the extended problem by computer-assisted means.